The dilution-resistant effect of dark particles on cosmic neutrinos

Xun-Jie Xu / 许勋杰

Institute of High Energy Physics (IHEP)

Chinese Academy of Sciences (CAS)

https://xunjiexu.github.io/

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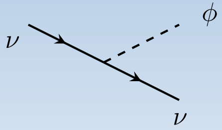
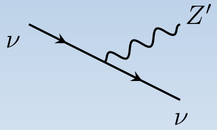
Everyone is talking about ν physics $\Rightarrow$ new physics ...

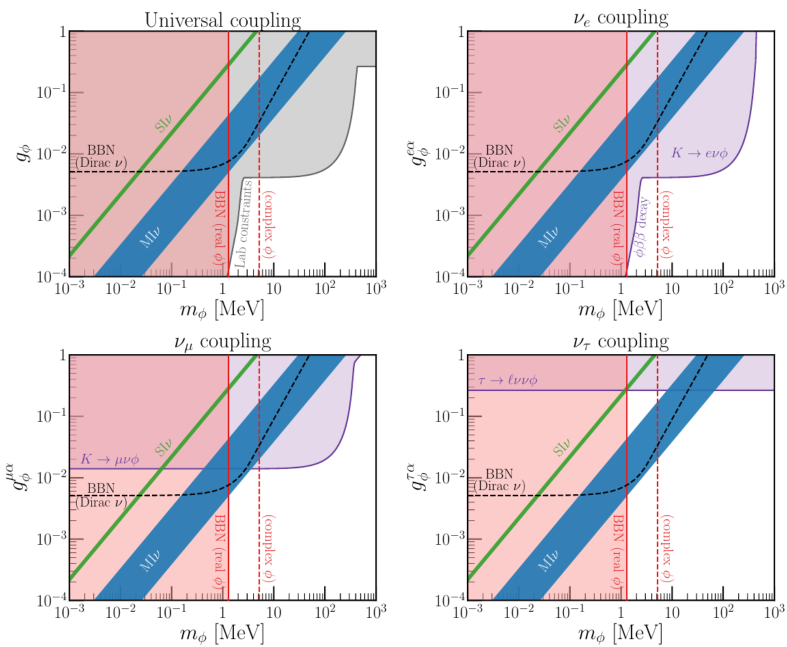
If neutrinos interact with a new particle, how can we probe it?

Simplest cases:



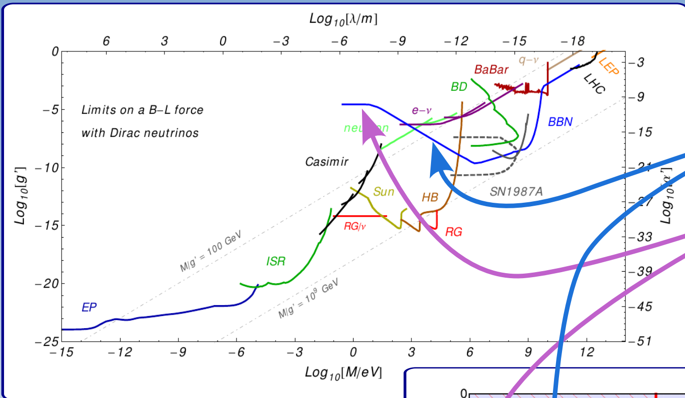


e.g. Majoron



N. Blinov, K. Kelly, G. Krnjaic, S. McDermott, PRL [1905.02727]

Constraints on $\nu\nu\phi$ or $\nu\nu Z'$



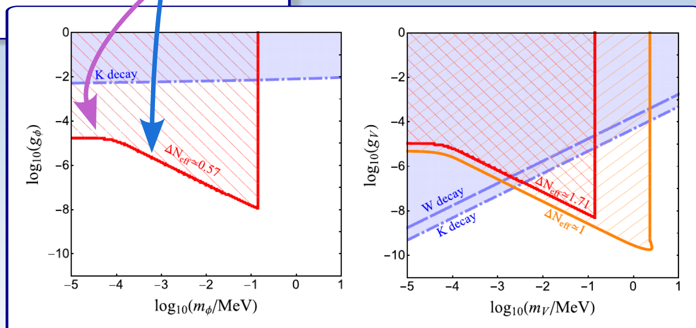
J. Heeck, PLB, [1408.6845]

$2\nu \leftrightarrow \phi$

$2\nu \leftrightarrow 2\phi$

BBN bound $\neq$ constant

- not a constant bound on g;
- not a constant bound on m.



G.-y. Huang, T. Ohlsson, and S. Zhou, PRD, [1712.04792]

Constraints on $\nu\nu\phi$ or $\nu\nu Z'$

processes	flavor dependence	bounds
$\pi^\pm$ decay	ν_e	$y < 1.3 \times 10^{-2}$
$K^\pm$ decay	ν_e, ν_μ	$y < 1.4 \times 10^{-2}$ (ν_e) or $< 3 \times 10^{-3}$ (ν_μ)
$\beta\beta$ decay	ν_e	$y < 3.4 \times 10^{-5}$
Z decay	all flavors	$y < 0.3$
BBN	all flavors	$y < 4.6 \times 10^{-6}$
CMB	all flavors	$y < 8.2 \times 10^{-7}$
SN1987A (energy loss)	all flavors	$y < 3 \times 10^{-7}$ or $2 \times 10^{-5} < y < 3 \times 10^{-4}$
SN1987A (deleptonization)	ν_e	$y < 2 \times 10^{-6}$

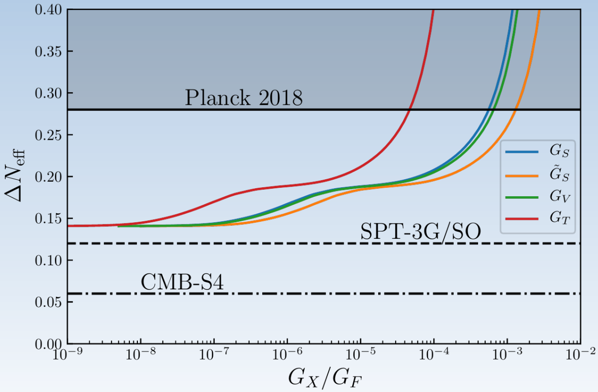
Smirnov, XJX, JHEP [2201.00939]

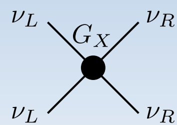
Among known bounds on very light ν-philic ϕ:

- CMB, SN, BBN are the strongest
  - ... but SN bounds ...
- Lab. bounds are often flavor-dependent (except for Z decay)

Cosmological constraints on light particles

- Why so stringent?
  - so stringent that many people don't even dare to go below MeV.
- robust?
  - yes! simply estimate the entropy (see next slides)
  - even Dirac neutrinos can be ruled out, if ν_R-SM int. is not too weak.





Luo, Rodejohann, XJX, JCAP, [2005.01629]

Simple exercise: let's derive $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$, 1.9K vs. 2.7K

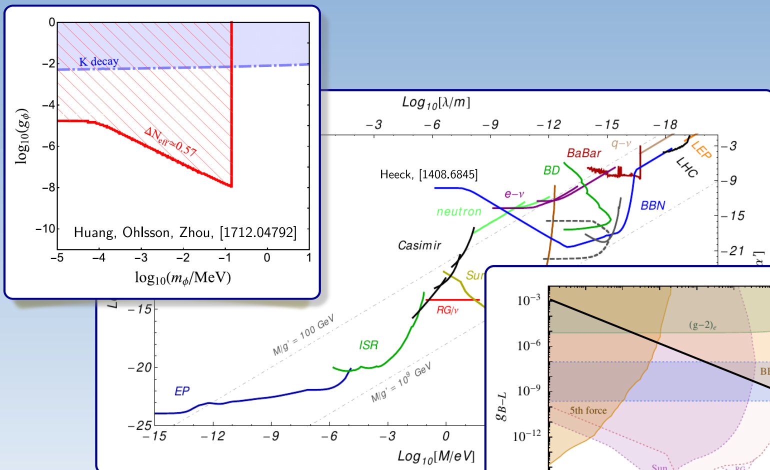
- just a game of entropy conservation
- let's derive it on a black board ...
  - $T_{\gamma 1}^3 \left(2 + 4 \times \frac{7}{8}\right) \to T_{\gamma 2}^3 (2+0)$, $T_{\nu 1}^3 \left(6 \times \frac{7}{8}\right) \to T_{\nu 2}^3 \left(6 \times \frac{7}{8}\right)$
- ..., which is why $N_{\rm eff}$ is defined as

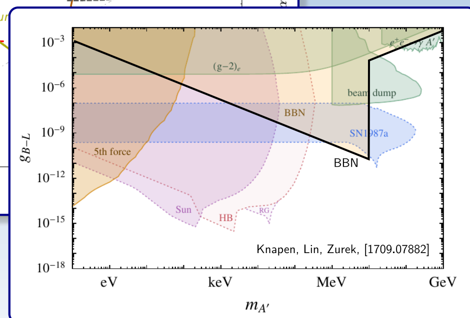
$$N_{\rm eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\left(\frac{\rho_{\rm rad} - \rho_\gamma}{\rho_\gamma}\right)$$

- SM prediction: $N_{\rm eff} = 3.045$; Planck 2018: $N_{\rm eff} = 2.99 \pm 0.17$;
- Any new entropy injection $\Rightarrow \Delta N_{\rm eff}$

Now, back to those CMB/BBN bounds on $\nu\nu\phi$ or $\nu\nu Z'$ ...

Why smaller $m \Rightarrow$ weaker bounds?

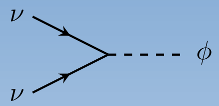




Answer: production rate suppressed by small m

... production rate suppressed by small m, why?

Simplest process for ϕ production:



- what if $m_\phi \leq m_\nu$?
  - kinematically forbidden (at least ϕ cannot decay to 2ν)
  - if decay is forbidden, inverse decay is also forbidden
- what if $m_\nu = 0$ and $m_\phi \to 0$?
  - must be suppressed by m_ϕ

Quantitatively ...

$$\frac{dn_\phi}{dt} + 3Hn_\phi = C_{\text{prod.}} - C_{\text{depl.}} \,,$$

$$C_{\text{prod.}} = \frac{y^2 m_\phi^3}{32\pi^3} T_\nu K_1\left(\frac{m_\phi}{T_\nu}\right)$$

indeed suppressed by m_ϕ

$H \gg \Gamma$ or $H \ll \Gamma$?

$$\frac{dn_\phi}{dt} + 3Hn_\phi = C_{\text{prod.}} - C_{\text{depl.}}\,,$$

$$C_{\text{prod.}} = \frac{y^2 m_\phi^3}{32\pi^3} T_\nu K_1\left(\frac{m_\phi}{T_\nu}\right) \sim \frac{y^2}{32\pi^3} \times \begin{cases} m_\phi^2 T_\nu^2 & (T_\nu \gg m_\phi) \\ \cdots e^{-m_\phi/T} & (T_\nu \gg m_\phi) \end{cases}$$

Define

$$\Gamma \equiv \frac{C_{\text{prod.}}}{n_\phi^{\text{eq}}}$$

Meaning: the probability of a ϕ particle being produced in a volume ($\supset 1\nu$) per unit time

Behavior: $$\Gamma \approx \frac{y^2 m_\phi}{32\pi} \begin{cases} m_\phi/T_\nu & (T_\nu \gg m_\phi) \\ 1 & (T_\nu \ll m_\phi) \end{cases}$$

The well-known criterion

In equilibrium: $H \ll \Gamma$
Not in Equilibrium: $H \gg \Gamma$

Fast reaction vs. fast expansion

Question: when is ϕ in equilibrium with ν?

$H \gg \Gamma$ or $H \ll \Gamma$?

Question: when is ϕ in equilibrium with ν?

$$\Gamma \approx \frac{y^2 m_\phi}{32\pi} \begin{cases} m_\phi / T_\nu & (T_\nu \gg m_\phi) \\ 1 & (T_\nu \ll m_\phi) \end{cases}$$

at high T_ν, the production is suppressed, while $H \propto T_\nu^2$

$\Gamma \ll H$

Physically, what does the suppression mean?

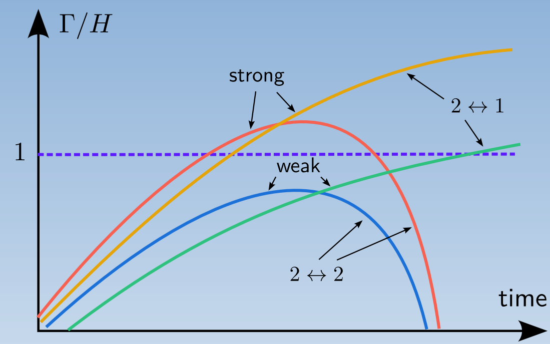
- again, think about the inverse process: if T_ν is very high, ϕ moves very rapidly
  - ultra-relativistic particles can hardly decay if E/m is too high
- slow decay $\Leftrightarrow$ slow inverse decay

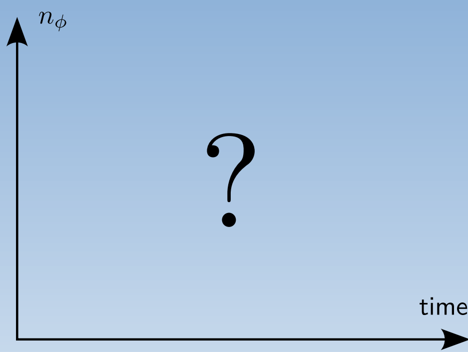
Moreover, written in term of the decay rate (at rest) Γ_0:

$$\Gamma \sim \Gamma_0 \begin{cases} m_\phi / T_\nu & (T_\nu \gg m_\phi) \\ 1 & (T_\nu \ll m_\phi) \end{cases}$$

essentially just the relativistic decay rate!

$\Gamma/H \ll 1$ at high T is quite common ...





A biased citation:

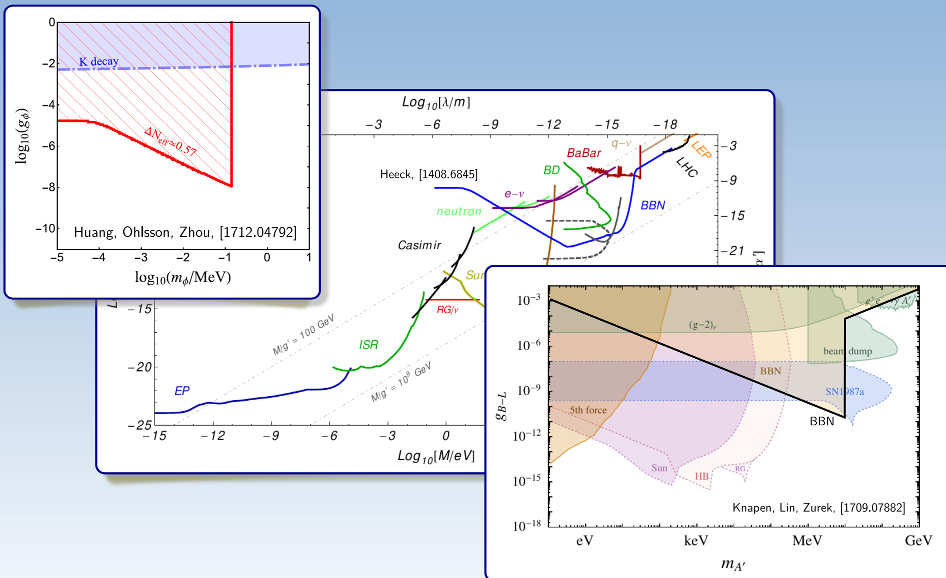
Freeze-In Production of FIMP Dark Matter #1

Lawrence J. Hall (UC, Berkeley and LBL, Berkeley and Tokyo U., IPMU), Karsten Jedamzik (Montpellier U.), John March-Russell (Oxford U., Theor. Phys.), Stephen M. West (Royal Holloway, U. of London and Rutherford) (Nov, 2009)

Published in: *JHEP* 03 (2010) 080 • e-Print: 0911.1120 [hep-ph]

pdf DOI cite claim reference search 1,126 citations

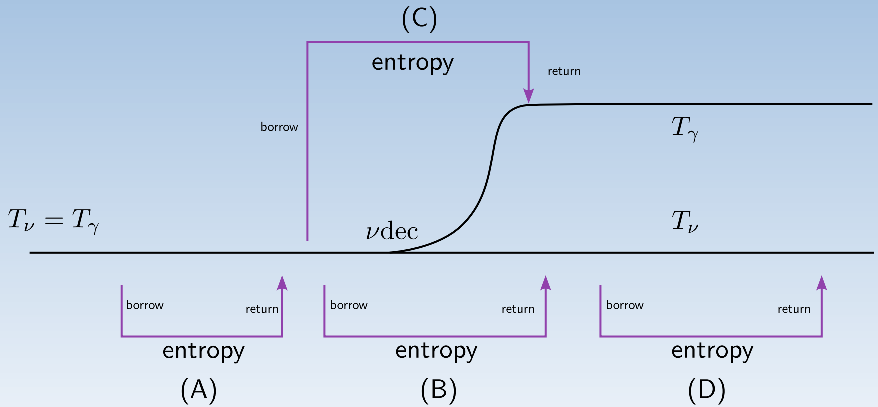
Now, can you understand the results in these papers?



... end of the pedagogical introduction.

Let's start to explore an interesting effect ...

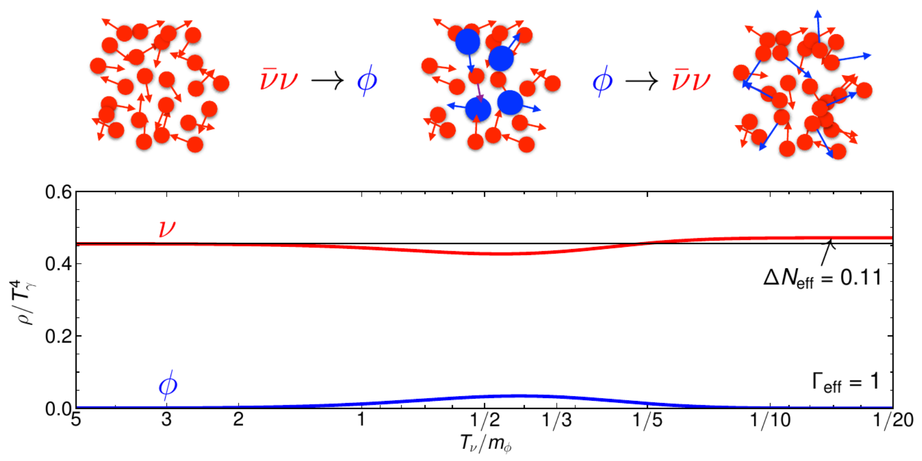
- In the sub-MeV regime, $m_\phi \downarrow \Rightarrow n_\phi @ \nu\mathrm{dec} \downarrow \Rightarrow$ entropy injection $\downarrow \Rightarrow \Delta N_{\rm eff} \downarrow$
- What if $n_\phi @ \nu\mathrm{dec} = 0$? corresponding to (D) below
  - $\Rightarrow \Delta N_{\rm eff} = 0$?
  - No! dilution-resistant effect (the subject of this talk)



Dilution-resistant effect — what is it?

- $N_{\rm eff}$ is changed even if the entire process is after $\nu_{\rm dec}$;
- already noticed previously

M. Escudero, JCAP, [2001.04466].



Let's think about the Boltzmann equations:

$$\frac{dn}{dt} + 3Hn = C^{(n)}_{\rm prod.} - C^{(n)}_{\rm depl.}\,, \qquad (1)$$

$$\frac{d\rho}{dt} + 3H\left(\rho + P\right) = C^{(\rho)}_{\rm prod.} - C^{(\rho)}_{\rm depl.}\,, \qquad (2)$$

$$H = a^{-1}\frac{da}{dt}$$

$$\frac{dn}{dt} + 3Hn = a^{-3}\frac{d\left(na^3\right)}{dt}$$

Eq. (1) $\Rightarrow$

$$\frac{d\left(na^3\right)}{da} = \frac{a^2}{H}\left[C^{(n)}_{\rm prod.} - C^{(n)}_{\rm depl.}\right]. \qquad (3)$$

Eq. (2) $\Rightarrow$

$$\frac{d\left(\rho a^4\right)}{da} = \frac{a^3}{H}\left[C^{(\rho)}_{\rm prod.} - C^{(\rho)}_{\rm depl.}\right] + a^3\left(\rho - 3P\right). \qquad (4)$$

- energy in ρa^4 not conserved
- if $\rho > 3P$, energy in ρa^4 $\uparrow$
- for γ & ν:
  - $\rho = 3P$
  - energy conserv.
- for ϕ with $m \neq 0$
  - $\rho > 3P$

Sum eqs for ν and ϕ (or Z') $\Rightarrow$

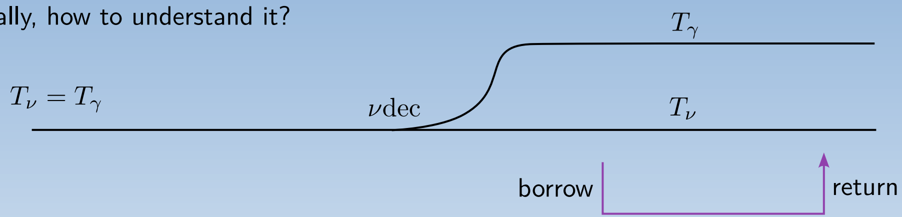
$$\frac{d\left(n_\nu a^3\right)}{da} + \frac{d\left(n_{Z'} a^3\right)}{da} = 0\,. \qquad (5)$$

$$\frac{d\left(\rho_\nu a^4\right)}{da} + \frac{d\left(\rho_{Z'} a^4\right)}{da} = a^3\left(\rho_{Z'} - 3P_{Z'}\right). \qquad (6)$$

$$\frac{d\left(\rho_{\text{total}} a^4\right)}{da} = a^3\left(\rho_{Z'} - 3P_{Z'}\right).$$

This is the dilution-resistant effect!

Physically, how to understand it?



energy stored in ϕ particles $\begin{cases} \text{stored in } m \\ \text{stored in } p \end{cases}$ universe expands $\begin{cases} m \to m \\ p \to p/a \end{cases}$

$\rho \propto a^{-3}$ vs $\rho \propto a^{-4}$

If you store energy in m, it will be less diluted by expansion

乱世买黄金 (buy gold during inflation)

Cosmological investment: 乱世买黄金 (buy gold during inflation)

$$\text{energy stored in } \phi \text{ particles} \begin{cases} \text{stored in } m \\ \\ \text{stored in } p \end{cases} \xrightarrow{\text{universe expands}} \begin{cases} m \to m \\ \\ p \to p/a \end{cases}$$

Cosmological investment: buy m, sell it later, profit$=\Delta(\rho a^4)$

- m — gold
- profit — earn money

Profit:

$$\Delta\left(\rho_{\text{total}} a^4\right) = \int a^3 \left(\rho - 3P\right) da \propto \int m \times \boxed{\cdots}\, da$$

Cosmological investment: 乱世买黄金 (buy gold during inflation)

How to compute the profit?

Method 1: Purely numeric. Solve all Boltzmann equations numerically to obtain the final ρ_ν

Method 2: Analytically estimate the integral

$$\text{Profit:}\quad \Delta\left(\rho_{\rm total}a^4\right) = \int a^3\left(\rho - 3P\right)da \propto \int m \times \boxed{\cdots}\, da$$

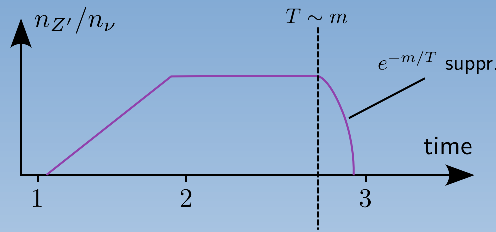
Method 3: Conservation of na^3, sa^3 and ρa^4 in their respective valid ranges

- Conservation of particle numbers (na^3).
  - always conserved if only $2 \leftrightarrow 1$ or $2 \to 2$ is present.
- Conservation of energy (ρa^4).
  - conserved if all particles are ultra-relativistic.
- Conservation of entropy (sa^3).
  - ... a subtle issue

Three methods $\Rightarrow$ same results [S.-P. Li, XJX, JCAP, 2307.13967]

Method 3: Conservation of na^3, sa^3 and ρa^4 in their respective valid ranges

$$n_1 a_1^3 = n_2 a_2^3 = n_3 a_3^3\,, \quad (1)$$
$$\rho_1 a_1^4 = \rho_2 a_2^4\,, \quad (2)$$
$$s_2 a_2^3 = s_3 a_3^3\,, \quad (3)$$



Eqs. (1)+(2) ⇒

$$n_{\mathsf{FD}}(T_1, 0) a_1^3 = \left[n_{\mathsf{FD}}(T_2, \mu_2) + n_{\mathsf{BE}}(T_2, 2\mu_2)\right] a_2^3\,,$$
$$2\rho_{\mathsf{FD}}(T_1, 0) a_1^4 = \left[2\rho_{\mathsf{FD}}(T_2, \mu_2) + \rho_{\mathsf{BE}}(T_2, 2\mu_2)\right] a_2^4\,,$$

$$n_{\mathsf{FD/BE}}(T,\mu) \equiv \int \frac{1}{e^{(p-\mu)/T} \pm 1} \frac{d^3p}{(2\pi)^3} = \mp \frac{T^3}{\pi^2} \mathsf{Li}_3(\mp e^{\mu/T}),$$
$$\rho_{\mathsf{FD/BE}}(T,\mu) \equiv \int \frac{p}{e^{(p-\mu)/T} \pm 1} \frac{d^3p}{(2\pi)^3} = \mp \frac{3T^4}{\pi^2} \mathsf{Li}_4(\mp e^{\mu/T}).$$

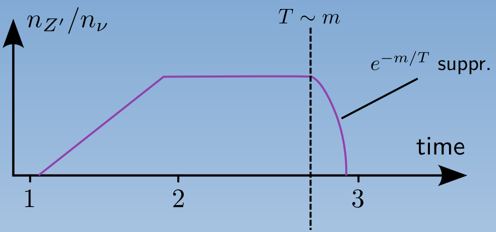
$$\cdots T_1^3 \mathsf{Li}(\cdots \mu_1 \cdots T_1) = \cdots T_2^3 \mathsf{Li}(\cdots \mu_2 \cdots T_2)$$
$$\cdots T_1^3 \mathsf{Li}(\cdots \mu_1 \cdots T_1) = \cdots T_2^3 \mathsf{Li}(\cdots \mu_2 \cdots T_2)$$

Li (polylog) functions are like log functions

— e.g. how to solve $x^2 \log(x) = 1$?

Solution: $(T_2,\ \mu_2) = (1.208, -1.166)\, T_1 \frac{a_1}{a_2}$

Method 3: Conservation of na^3, sa^3 and ρa^4 in their respective valid ranges



From eqs connecting "1" to "2",

Solution: $(T_2,\ \mu_2) = (1.208, -1.166)\, T_1 \frac{a_1}{a_2}$

From eqs connecting "2" to "3", similar:

Solution: $(T_3,\ \mu_3) = (1.092, -0.3133)\, T_1 \frac{a_1}{a_3}$

$$\Delta N_{\text{eff}} = 3\left[\frac{\rho_{\nu 3} a_3^4}{\rho_{\nu 1} a_1^4} - 1\right] = 0.242\,.$$

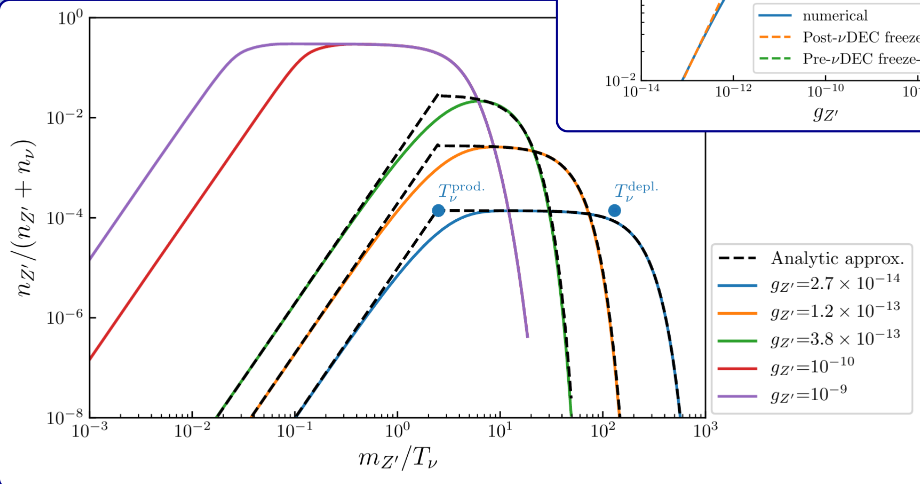
for the vector case (Z')

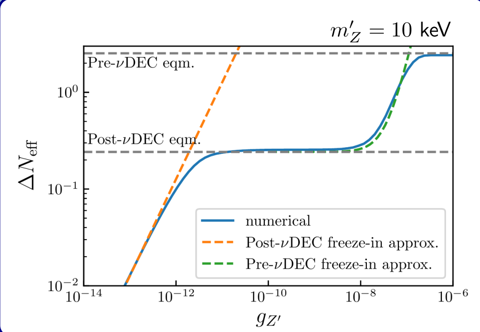
More generally, ...

	scalar	vector
Post-νDEC equilibrium	$\Delta N_{\text{eff}} = 0.118$	$\Delta N_{\text{eff}} = 0.242$
Pre-νDEC equilibrium	$\Delta N_{\text{eff}} = 0.794$	$\Delta N_{\text{eff}} = 2.53$
Pre-νDEC equilibrium (strong couplings)	$\Delta N_{\text{eff}} = 0.785$	$\Delta N_{\text{eff}} = 2.48$

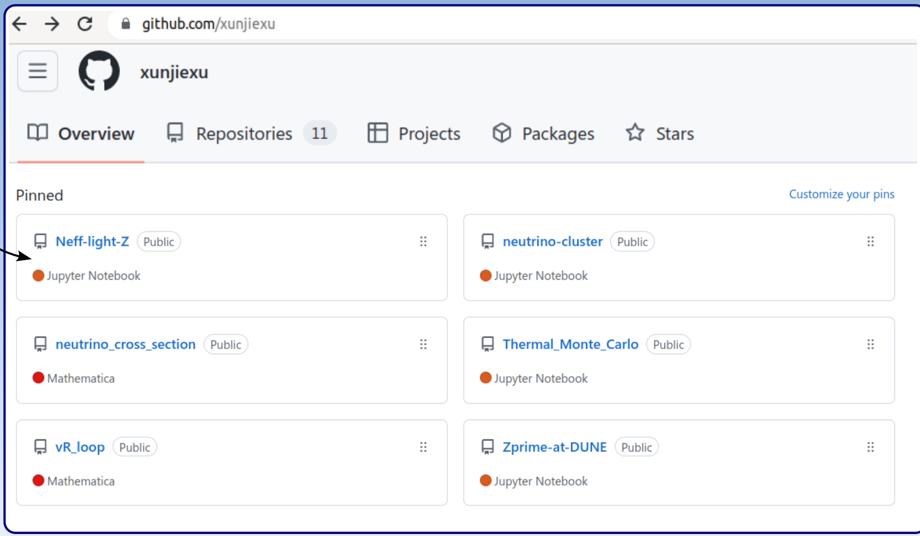
Alternatively, we can solve all Boltzmann equations numerically to obtain the final ρ_ν

Results agree well the analytic

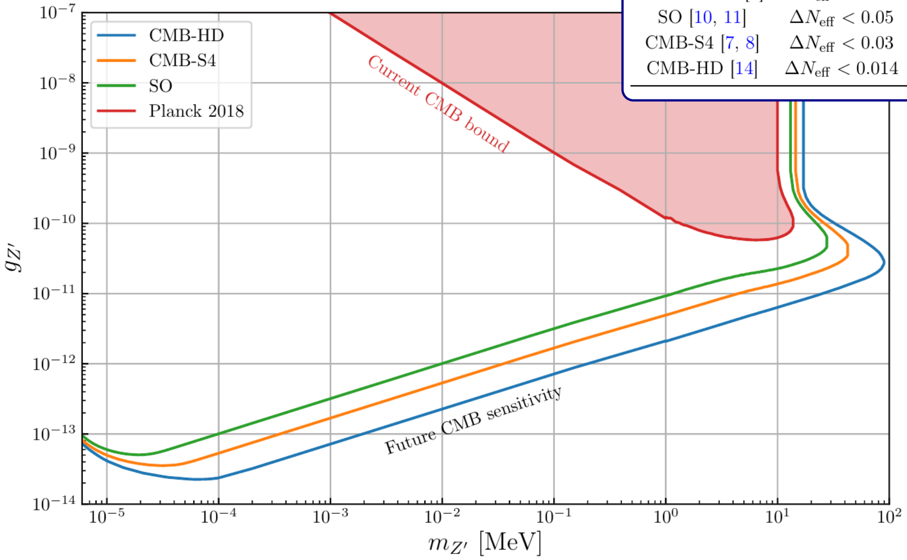




Code publicly available at https://github.com/xunjiexu



Future experiments ⇒ $\Delta N_{\rm eff}$ improved by orders of mangitude



	$\Delta N_{\rm eff}$ (1σ)	$\Delta N_{\rm eff}$ (2σ)
Planck 2018 [1]	$\Delta N_{\rm eff} < 0.115$	$\Delta N_{\rm eff} < 0.285$
SO [10, 11]	$\Delta N_{\rm eff} < 0.05$	$\Delta N_{\rm eff} < 0.1$
CMB-S4 [7, 8]	$\Delta N_{\rm eff} < 0.03$	$\Delta N_{\rm eff} < 0.06$
CMB-HD [14]	$\Delta N_{\rm eff} < 0.014$	$\Delta N_{\rm eff} < 0.028$

Future experiments $\Rightarrow \Delta N_{\rm eff}$ improved by orders of mangitude

	$\Delta N_{\rm eff}$ (1σ)	$\Delta N_{\rm eff}$ (2σ)
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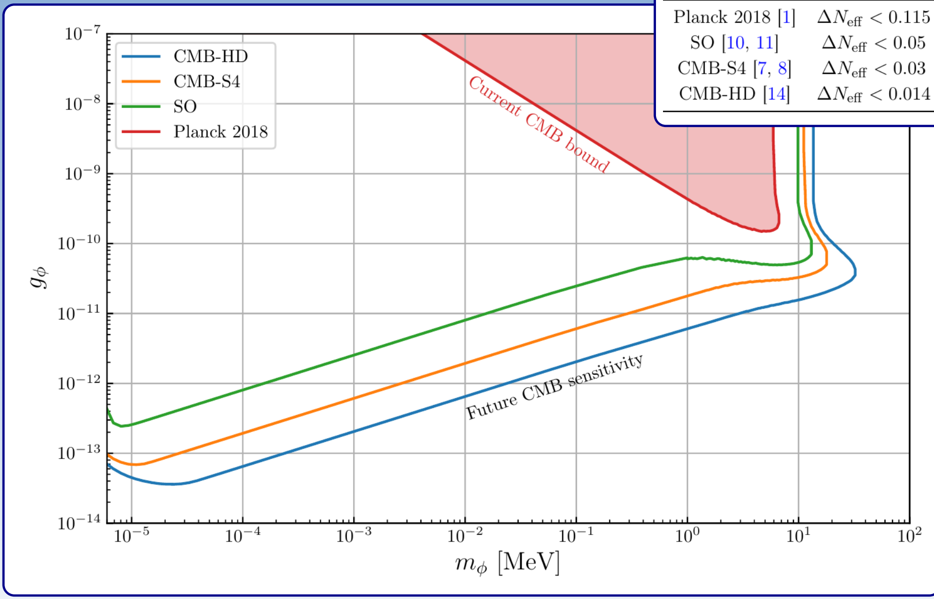
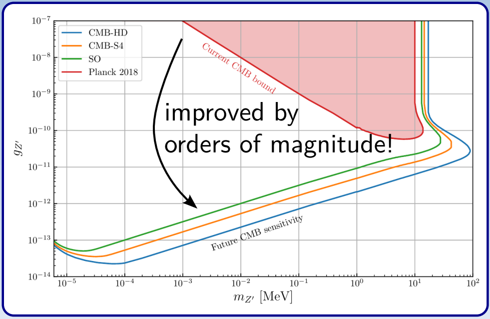


Pedagogical introduction:

- Why $T_\gamma/T_\nu = (4/11)^{1/3}$?
- How to estimate $\Delta N_{\rm eff}$ due to new physics?
- Why $\Delta N_{\rm eff}$ decreases as m creases?

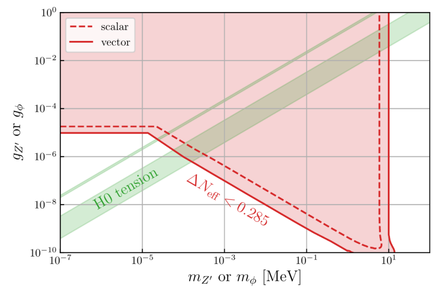
A bit more advanced (what if $n_{\phi/Z'} \to 0$ at νdec?)

- Dilution-resistant effect
  - Cosmological investment: $m =$ gold
  - $\Delta N_{\rm eff} = 0.118$ (scalar) or 0.242 (vector)
- current exp almost able to probe (Planck 2018, $\Delta N_{\rm eff} < 0.285$)
- future exp can reach it $\Rightarrow$ bounds on g improved by orders of magnitude!

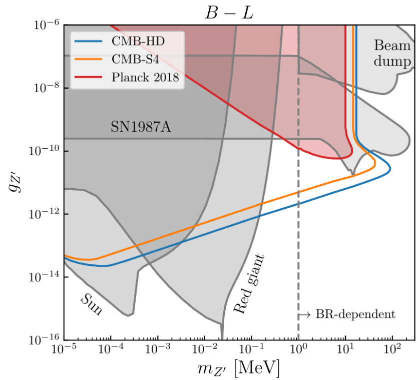


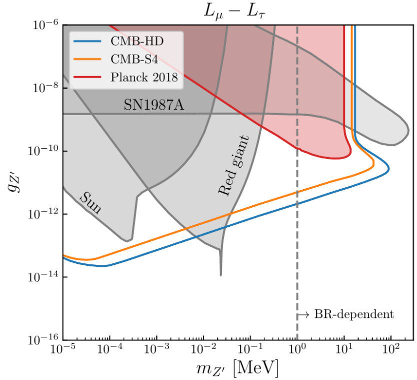
Backup

Hubble tension



Specific models





Why is there a nose?