Wolfenstein, Yukawa, and Coulomb Potentials

— ν oscillation in matter with heavy, light, and ultralight mediators

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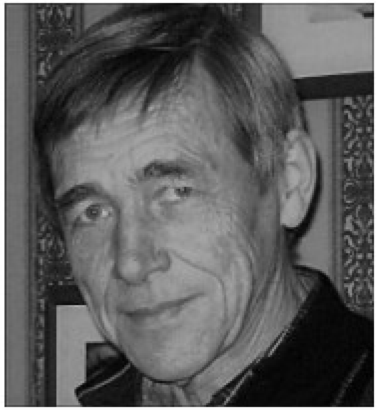
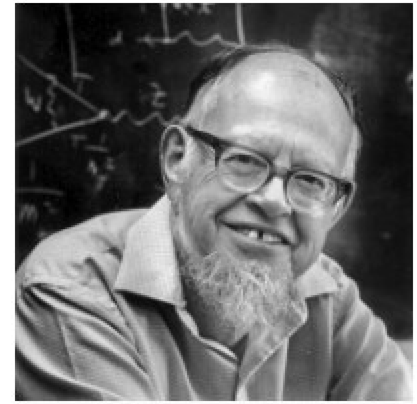
Talk based on: 1909.07505, in collaboration with Alexei Smirnov.
IHEP Seminar, May 21, 2020.

The Mikheyev-Smirnov-Wolfenstein effect

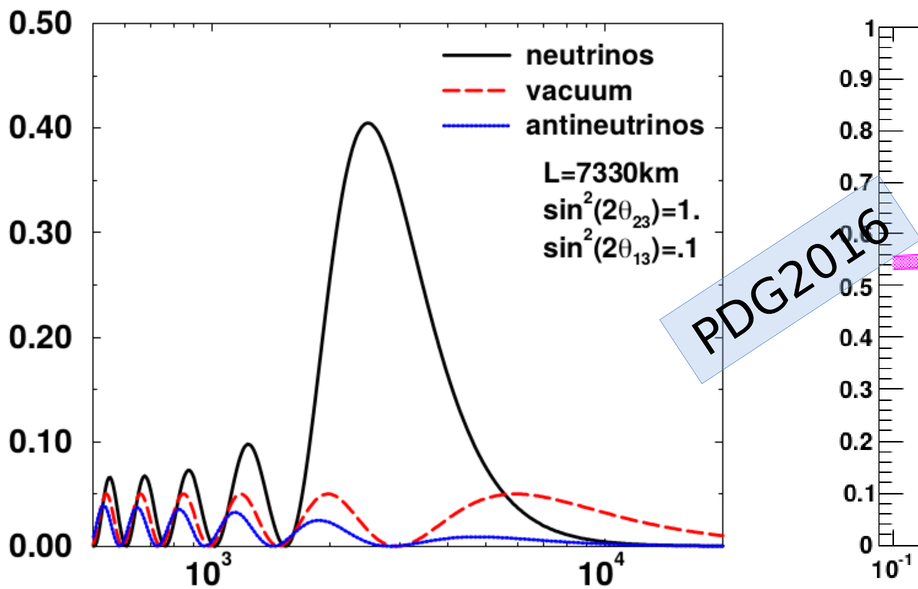


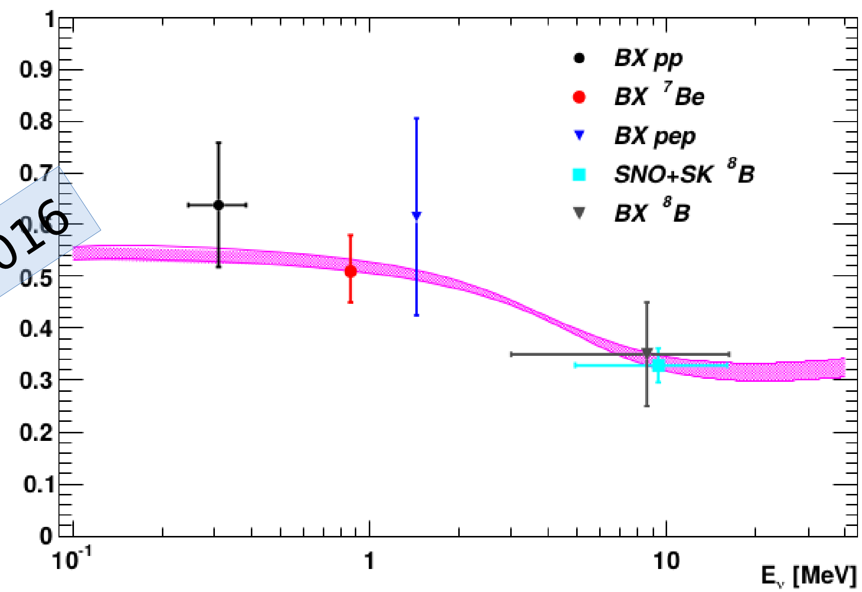
S. Mikheyev
(1940-2011)

A. Smirnov
(1951-)



L. Wolfenstein
(1923-2015)





The Mikheyev-Smirnov-Wolfenstein effect

How do neutrinos oscillate?

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = H\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

The matter (MSW) effect

$$H = \frac{1}{2E}U\begin{pmatrix}m_1^2 & & \\ & m_2^2 & \\ & & m_3^2\end{pmatrix}U^\dagger + \begin{pmatrix}V & & \\ & 0 & \\ & & 0\end{pmatrix}$$

The Wolfenstein potential:

$$V = \sqrt{2}G_F n_e$$

The Mikheyev-Smirnov-Wolfenstein effect

The Wolfenstein potential:

$$V = \sqrt{2} G_F n_e$$

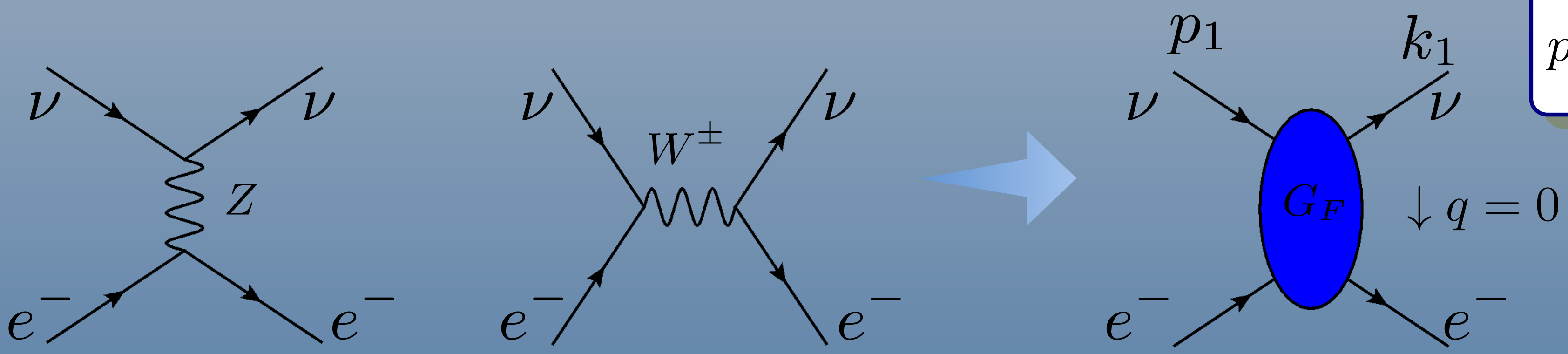
Questions:

- why such a form?
- what if ν's have new interactions?
- is it always valid?

The first question: why such form?

$$V = \sqrt{2} G_F n_e$$

MSW effect $\Leftarrow$ coherent forward scattering



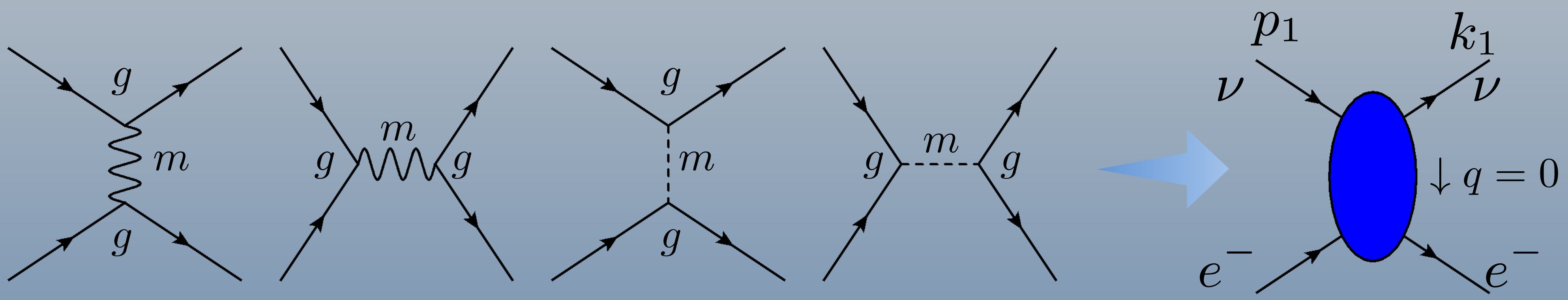
$p_1 = k_1$

"forward" means: zero momentum transfer from ν to e^-

$\Rightarrow$ the scattering amplitude $\propto G_F$

"coherent" $\Rightarrow$ adding amplitudes coherently $\Rightarrow V \propto n_e$

The 2nd question: what if ν's have new interactions?

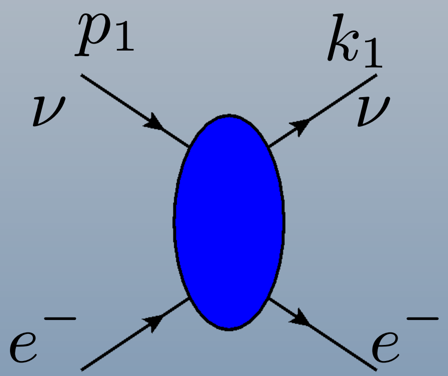


$$V = n_e \frac{g^2}{m^2}$$

Due to $p_1 = k_1$, all can be reduced to effective contact interactions.

The Mikheyev-Smirnov-Wolfenstein effect

What if ν's have new interactions?



Due to $p_1 - k_1 = 0$, all can be reduced to effective contact interactions.

$$V = n_e \frac{g^2}{m^2}$$

Example:

SM: $\mathcal{L} \supset G_F \, \overline{\nu}_\alpha \gamma^\mu \nu_\beta \, \overline{e} \gamma_\mu e$

$V = \sqrt{2} G_F n_e$

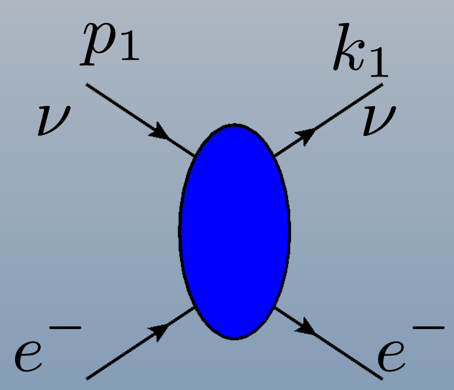
$Z' \Rightarrow$ NSI: $\mathcal{L} \supset G_F \, \epsilon_{\alpha\beta} \, \overline{\nu}_\alpha \gamma^\mu \nu_\beta \, \overline{e} \gamma_\mu e$

$V = \sqrt{2} G_F n_e \epsilon_{\alpha\beta}$

$$H = \frac{1}{2E} U \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} U^\dagger + G_F n_e \begin{pmatrix} \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ . & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ . & . & \epsilon_{\tau\tau} \end{pmatrix}$$

Oscillation possible even if $m = 0$. [L. Wolfenstein, PRD, 1978]

The 2nd question: what if ν's have new interactions?



Due to $p_1 = k_1$, all can be reduced to effective contact interactions.

$$V = n_e \frac{g^2}{m^2}$$

But, for scalar interactions, same V, different effect!

Vector NSI: $\mathcal{L} \supset G_F \, \epsilon_{\alpha\beta} \, \overline{\nu}_\alpha \gamma^\mu \nu_\beta \, \overline{e} \gamma_\mu e$

Scalar NSI: $\mathcal{L} \supset G_F \, \epsilon_{\alpha\beta} \, \overline{\nu}_\alpha \mathbf{1} \nu_\beta \, \overline{e} \mathbf{1} e$

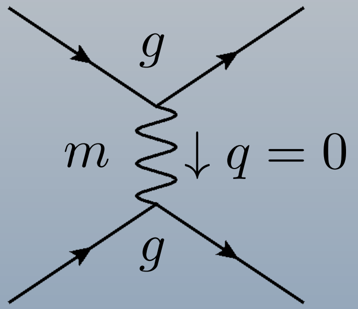
$$V = \sqrt{2} G_F n_e \epsilon_{\alpha\beta}$$

Vector interactions: $H \to H + V$

Scalar interactions: $m_\nu \to m_\nu + V$

Suppressed by m_ν / E_ν
Well known
I'll talk more later

The third question:



$\Rightarrow$

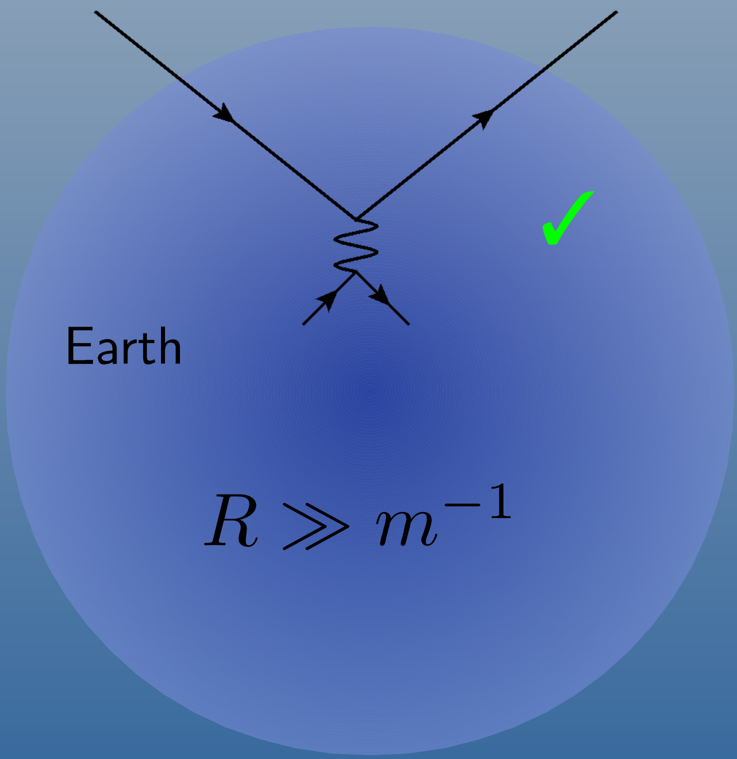
$$V = n_e \frac{g^2}{m^2}$$

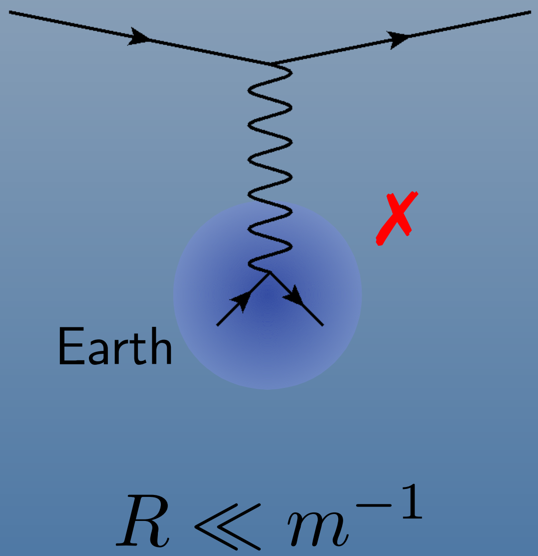
Note that $\frac{i}{q^2 - m^2} = -\frac{i}{m^2}$

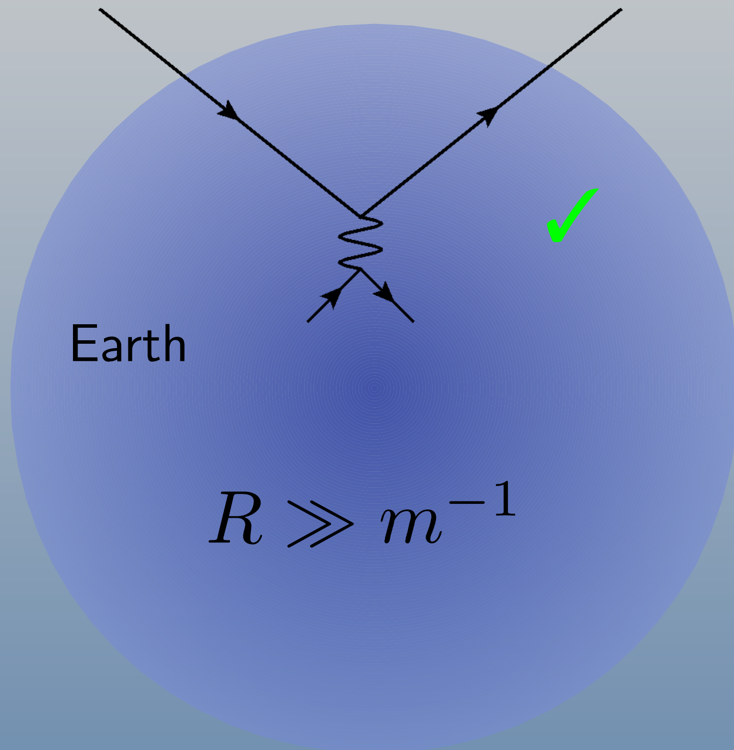
Q: always valid?

If $m \to 0$, $V \to \infty$?

A: No!

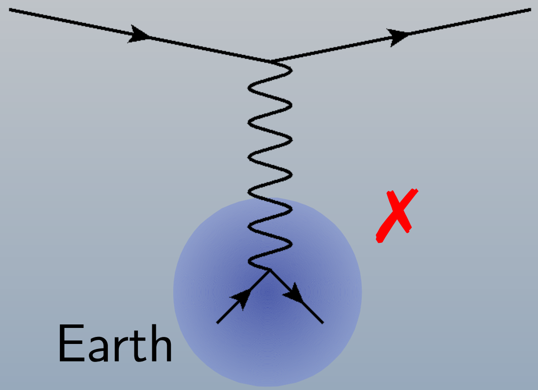






$$R \gg m^{-1}$$

$$V = n_e \frac{g^2}{m^2}$$



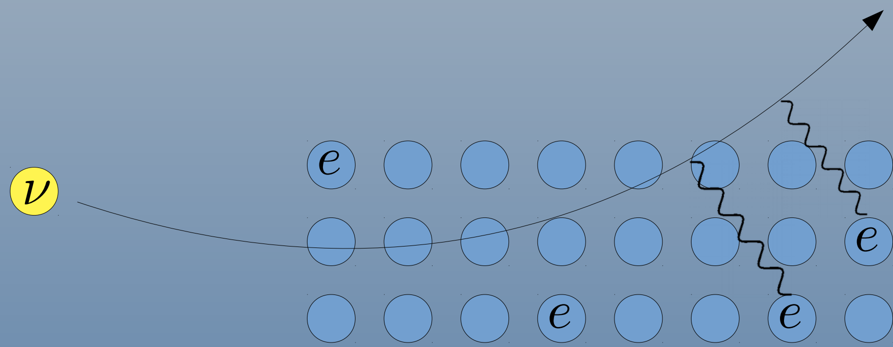
$$R \ll m^{-1}$$

$$V = ???$$

The main question of this talk: How is V modified?

Let's re-derive the MSW effect!

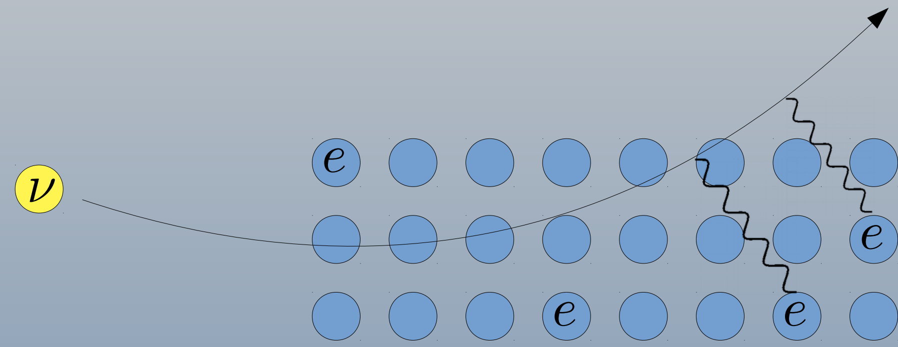
- Consider a simple system, only neutrino (ν) + electron (e) + vector boson (A^μ)
- Electrons are fixed by external forces, e.g., the electromagnetic (EM) force
- Neutrinos propagate through these electrons
- There are interactions between ν and e, mediated by the vector boson A^μ.



$$\mathcal{L} \supset \overline{\nu} i \partial\!\!\!/ \nu - m_\nu \overline{\nu}\nu - g\overline{\nu} A\!\!\!/ \nu - g\overline{e} A\!\!\!/ e - \frac{m^2}{2} A^\mu A_\mu$$

$\mathcal{L} \Rightarrow$EOM (equation of motion) $\Rightarrow$ MSW effect

Let's re-derive the MSW effect!



$$\mathcal{L} \supset \overline{\nu} i \partial\!\!\!/ \nu - m_\nu \overline{\nu}\nu - g\overline{\nu} A\!\!\!/ \nu - g\overline{e} A\!\!\!/ e - \frac{m^2}{2} A^\mu A_\mu$$

$\mathcal{L} \Rightarrow$EOM (equation of motion) $\Rightarrow$ MSW effect

EOM:

Compare to
Dirac Equation:
$(i\partial\!\!\!/ + m)\psi = 0$
Maxwell Equation:
$\partial^2 A^\mu = j^\mu$

$$i\partial\!\!\!/ \nu - m_\nu \nu - g A\!\!\!/ \nu = 0,$$

$$\left[\partial^2 + m^2\right] A^\mu - g\overline{\nu}\gamma_\mu \nu - g\overline{e}\gamma_\mu e = 0$$

plus EOM of electrons $\cdots$

Let's re-derive the MSW effect!

EOM (equation of motion)

$$i\partial\!\!\!/\nu - m_\nu \nu - gA\!\!\!/\nu = 0,$$

$$\left[\partial^2 + m^2\right] A^\mu - g\bar{\nu}\gamma_\mu \nu - g\bar{e}\gamma_\mu e = 0$$

$$\text{plus EOM of electrons} \cdots$$

Next, solve EOM

the fields of ν, A^μ, e are determined by the solutions of EOM

$e \Rightarrow A^\mu \Rightarrow \nu$

- e field already known (fixed on the lattice)
- A^μ field almost determined by e field
- ν field to be determined by A^μ field

assuming: ν field $\ll$ e field

Let's re-derive the MSW effect!

EOM

$$i\partial\!\!\!/\nu - m_\nu \nu - gA\!\!\!/\nu = 0$$

$$\left[\partial^2 + m^2\right] A^\mu - g\bar{\nu}\gamma_\mu\nu - g\bar{e}\gamma_\mu e = 0$$

assuming: ν field $\ll$ e field

Physical meaning of $g\bar{e}\gamma_\mu e$:

see Peskin&Schroeder's textbook

- $J^i \equiv \bar{e}\gamma^i e$: electric current density (spatial)
- $n_e \equiv \bar{e}\gamma^0 e$: electron number density

charge conservation: $\partial_t n_e = \nabla \cdot J$

Usually electrons in matter are almost at rest $\Rightarrow J^i = 0$.
(average velocity $=0$)

So

$$\bar{e}\gamma^\mu e = (n_e,\ 0,\ 0,\ 0)$$

Let's re-derive the MSW effect!

EOM

$$i\partial\!\!\!/\nu - m_\nu \nu - g A\!\!\!/ \nu = 0$$

$$\left[\partial^2 + m^2\right] A^\mu - g\bar{\nu}\gamma_\mu\nu - g\bar{e}\gamma_\mu e = 0$$

$$\left[\partial^2 + m^2\right] A^\mu - g\bar{e}\gamma_\mu e = 0$$

$\bar{e}\gamma^\mu e = (n_e,\ 0,\ 0,\ 0)$
Hence
$A^\mu = (A^0,\ 0,\ 0,\ 0)$

$$\left[\partial^2 + m^2\right] A^0 - g n_e = 0$$

$\partial_t A^0 = 0$ (due to static charge distribtion $\partial_t n_e = 0$)

$$\left[-\nabla^2 + m^2\right] A^0 - g n_e = 0$$

Let's re-derive the MSW effect!

EOM

$$i\partial\!\!\!/\nu - m_\nu \nu - gA\!\!\!/\nu = 0$$

$$\left[\partial^2 + m^2\right] A^\mu - g\bar{e}\gamma_\mu e = 0$$

$$\left[-\nabla^2 + m^2\right] A^0 = g n_e$$

Given n_e, we can determine A^0. Then $A^\mu = (A^0, 0, 0, 0)$ is determined.

Given A^μ, how does ν propagate?
$(i\partial\!\!\!/ - gA\!\!\!/)\nu - m_\nu \nu = 0$

In the momentum space, $i\partial\!\!\!/ \to -p^\mu \gamma_\mu$ so:

$$p^\mu \to p^\mu + gA^\mu$$

$$E = p^0 \to E + V, \; V \equiv gA^0$$

Let's re-derive the MSW effect!

The master formula

$$\left[-\nabla^2 + m^2\right] A^0 = g n_e$$

- It's a differential equation.
- Given the distribution of electron density (n_e), we can always figure out A^0 by solving the equation.
- Physical meaning: $g n_e$ is the source that excites the A^0 field.
- $\left[-\nabla^2 + m^2\right] A^0 = g(n_{e1} + n_{e2} + n_{e3} + \cdots)$.

Let's re-derive the MSW effect!

Solve the differential equation

$$\left[-\nabla^2 + m^2\right] A^0 = g n_e$$

Spherical cows in a vacuum

There's a dairy farm. One day, the milk production was low. So the farmer called a physicist to help. The physicist then did a lot of calculations, and he said: "um, I have a solution, but it only works with spherical cows in a vacuum."

From Wikipedia: https://en.wikipedia.org/wiki/Spherical_cow

Let's re-derive the MSW effect!

Solve the differential equation

$$\left[-\nabla^2 + m^2\right] A^0 = g n_e$$

Let's assume a spherical cow:

$$n_e(r) = \begin{cases} 0 & (\text{for } r > R) \\ n_e & (\text{for } r \leq R) \end{cases},$$

the solution is

$$A^0(r) = \frac{g n_e}{m^2} F(r),$$

$$F(r) = \begin{cases} 1 - \frac{mR+1}{mr} e^{-mR} \sinh(mr) & (r \leq R) \\ \frac{e^{-mr}}{mr} \left[mR \cosh(mR) - \sinh(mR)\right] & (r > R) \end{cases}$$

Interesting limits of the spherical cow solution

- very compact cow ($R \to 0$ while $N_e \equiv \frac{4}{3}\pi R^3 n_e = \text{const.}$):

$$V(r) = \frac{g^2}{m^2} N_e \frac{e^{-mr}}{4\pi r}$$

$\Rightarrow$ the well-known Yukawa potential

- long-range interacting cow ($m \to 0$):

$$V(r) = g^2 n_e \times \begin{cases} \frac{3R^2 - r^2}{6} & (r \leq R) \\ \frac{R^3}{3}\frac{1}{r} & (r > R) \end{cases}$$

$\Rightarrow$ the well-known Coulomb potential

Interesting limits of the spherical cow solution

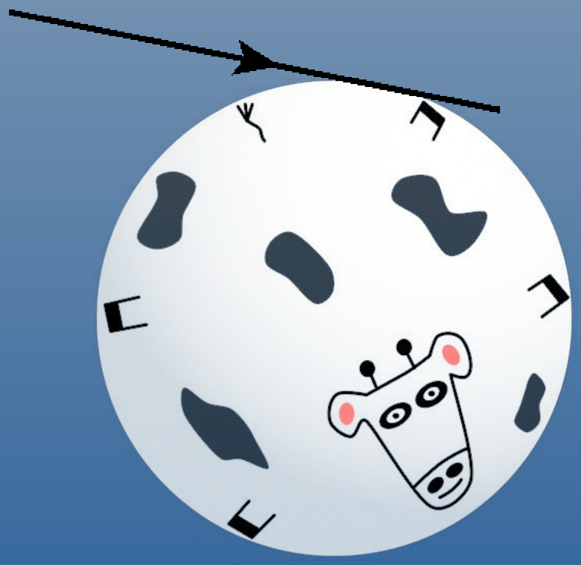
- contact-interacting cow ($m \to \infty$):

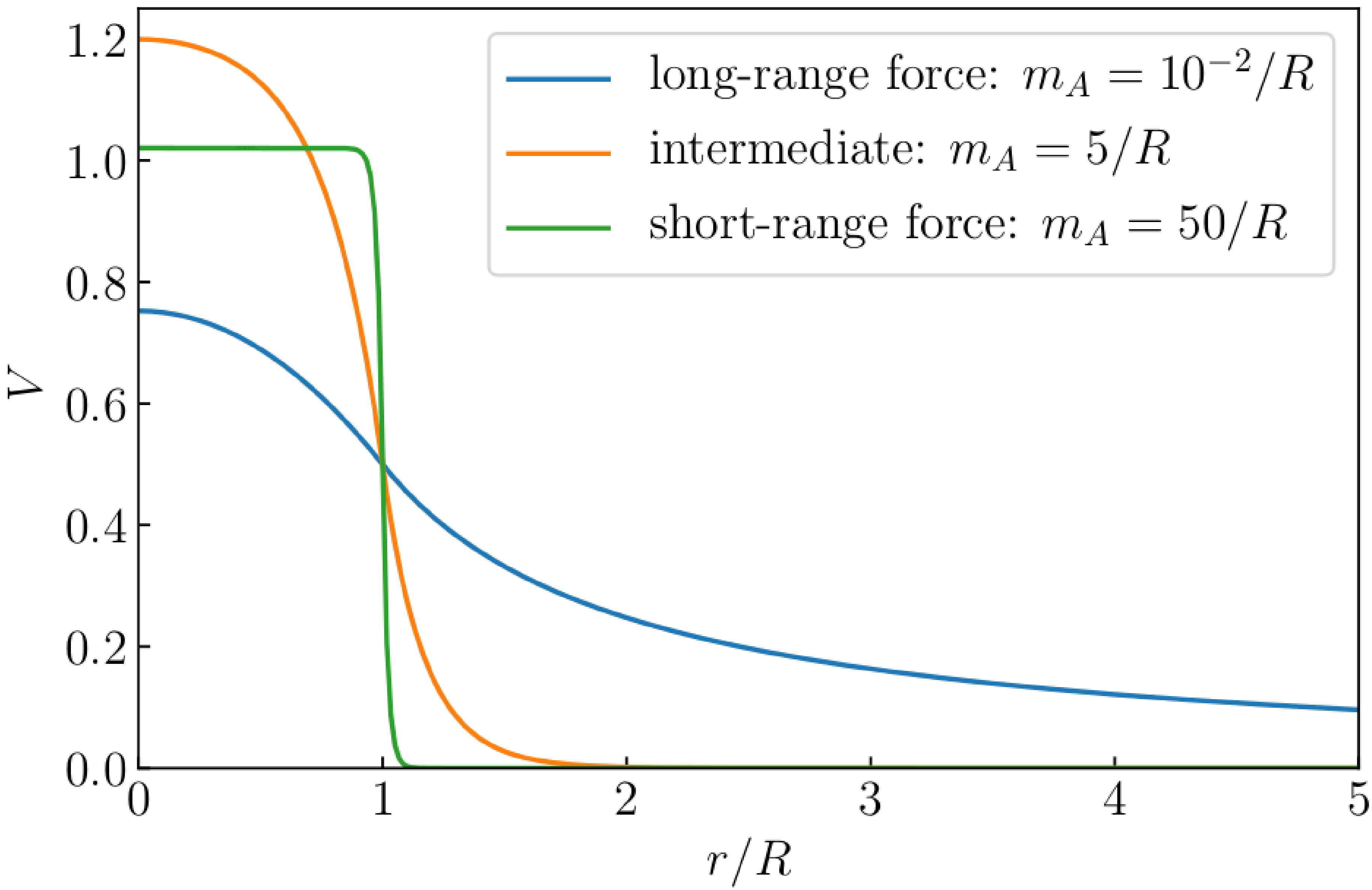
$$V(r) = \begin{cases} \frac{g^2}{m^2} n_e & (r \le R) \\ 0 & (r > R) \end{cases}$$

$\Rightarrow$ the standard Wolfenstein potential

- ν at cow's skin ($|R - r| \ll m^{-1} \ll R$):

$$V(r) = \frac{1}{2}\frac{g^2}{m^2} n_e$$





long-range force: $m_A = 10^{-2}/R$
intermediate: $m_A = 5/R$
short-range force: $m_A = 50/R$
V
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
r/R

Vector $\rightarrow$ Scalar

Vector $\to$ Scalar

$$\mathcal{L} \supset \overline{\nu} i \partial\!\!\!/ \nu - m_\nu \overline{\nu}\nu - g\overline{\nu} A\!\!\!/ \nu - g\bar{e} A\!\!\!/ e - \frac{m^2}{2} A^\mu A_\mu$$

$$i\partial\!\!\!/ \nu - m_\nu \nu - g A\!\!\!/ \nu = 0$$

$$\left[\partial^2 + m^2\right] A^\mu - g\bar{e}\gamma_\mu e = 0$$

$$\mathcal{L} \supset \overline{\nu} i \partial\!\!\!/ \nu - m_\nu \overline{\nu}\nu - g\overline{\nu}\phi\nu - g\bar{e}\phi e - \frac{m^2}{2}\phi^2$$

$$i\partial\!\!\!/ \nu - m_\nu \nu - g\phi\nu = 0$$

$$\left[\partial^2 + m^2\right]\phi - g\overline{e}e = 0$$

Vector $\rightarrow$ Scalar

$$\left[\partial^2 + m^2\right] A^\mu - g\bar{e}\gamma_\mu e = 0 \quad \Rightarrow \quad \left[\partial^2 + m^2\right] \phi - g\bar{e}e = 0$$

$$\left[-\nabla^2 + m^2\right] A^0 - g n_e = 0 \quad \Rightarrow \quad \left[-\nabla^2 + m^2\right] \phi - g n_e = 0$$

Same equation $\Rightarrow$ same solution!

But!

$$i\partial\!\!\!/\nu - m_\nu \nu - g A\!\!\!/\nu = 0 \quad \Rightarrow \quad i\partial\!\!\!/\nu - m_\nu \nu - g\phi\nu = 0$$

$$p^\mu \rightarrow p^\mu + gA^\mu$$

$$E \rightarrow E + gA^0$$

$$\Rightarrow \quad m_\nu \rightarrow m_\nu + g\phi$$

$$(i\partial\!\!\!/ - m_\nu - gA^\mu\gamma_\mu)\nu = 0 \quad \Rightarrow \quad (i\partial\!\!\!/ - m_\nu - g\phi)\nu = 0$$

$$E \to E + gA^0 \quad \Rightarrow \quad m_\nu \to m_\nu + g\phi$$

Vector interactions: $H \to H + V$, $V = gA^0$

Scalar interactions: $m_\nu \to m_\nu + V$, $V = g\phi$

$\Rightarrow$ very different effects in ν oscillation.

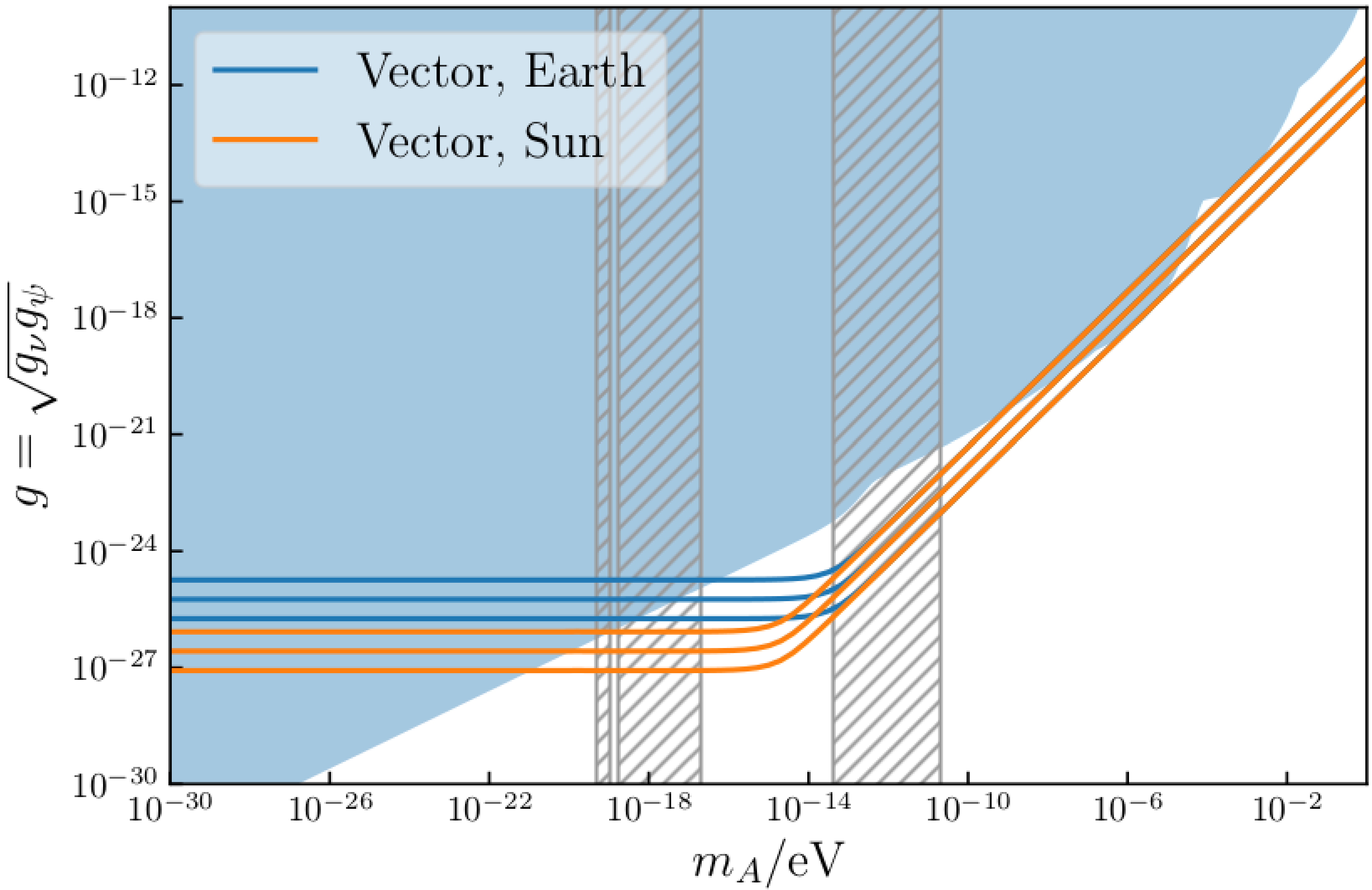
Comments on scalar interactions:

- Typically suppressed by m_ν/E_ν.

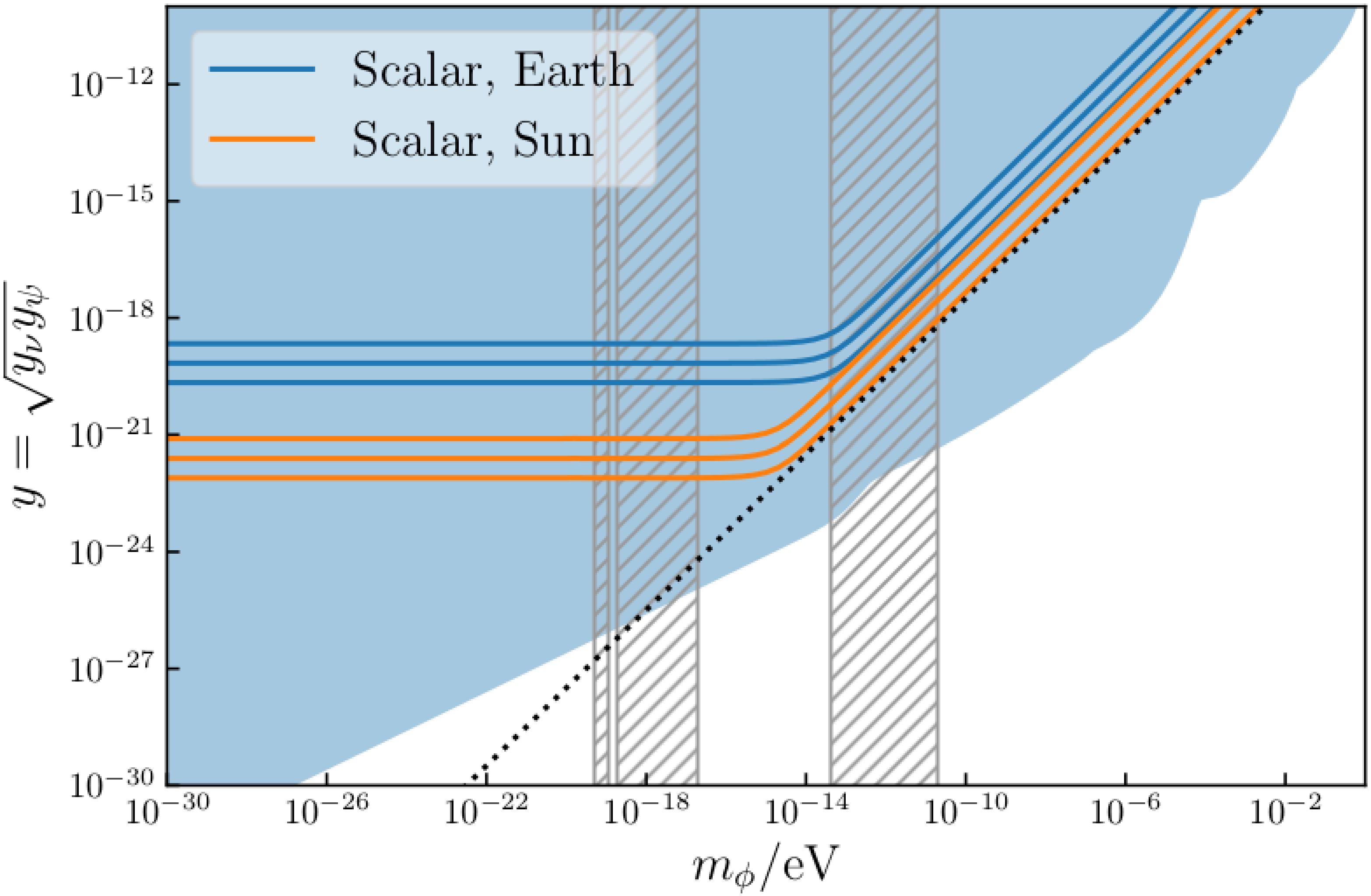
E.g. $m_\nu \sim 0.01$ eV, $E_\nu \sim 100$ MeV, $\Rightarrow m_\nu/E_\nu \sim 10^{-10}$

- Impossible to have any significant effect in any realistic experiments.
- at Earth: $m_\nu \to m_\nu + 0.01$ eV $\Rightarrow$ in SN: $m_\nu \to m_\nu + 100$ GeV $\Rightarrow$ no SN ν!

Vector effects, assuming $\frac{V}{V_{\rm SM}} = \{10^{-2},\ 10^{-1},\ 1\}$



Scalar effects, assuming $\delta m_\nu = \{10^{-3},\ 10^{-2},\ 10^{-1}\}$ eV



Summary

Take-home messages

1. This is a powerful cow

   Spherical cow in a vacuum
   $\Rightarrow$ Yukawa, Coulomb, Wolfenstein potentials.

$$\left[-\nabla^2 + m^2\right] A^0 = g n_e$$

2. Scalar matter effect doesn't work

   requires $10^{10} G_F$,
   excluded by, e.g., ν free streaming, $m_\nu \to m_\nu + \delta m$ in supernova.

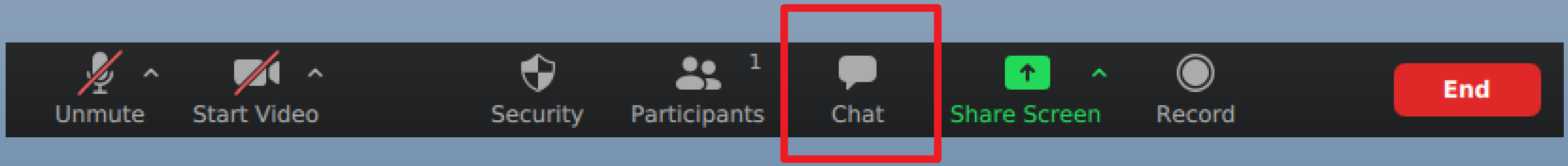
3. Vector matter effect works

   High energy: normal NSI
   Low energy: e.g. $m_A \in [2 \times 10^{-17}, 4 \times 10^{-14}]$ eV, $g \sim 10^{-25}$.

Backup

Tips for Zoom users:

1. In case of bad internet connection, if you have questions or comments during my talk, you can leave a message in the Zoom chat room.



2. You can download the pdf of this talk from Zoom.

3. you can draw on the screen!

Feel free to try these functions!